ENHANCING RECURSIVE CONSTRAINT BOUNDING FOR IMPROVED ROBUSTNESS, FUNCTIONALITY, AND FLEXIBILITY

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ABSTRACT

Modeling uncertainty in machining, caused by modeling inaccuracy, noise and process time-variability due to tool wear, hinders application of traditional optimization to minimize cost or production time. Process time-variability can be overcome by adaptive control optimization (ACO) to improve machine settings in reference to process feedback so as to satisfy constraints associated with part quality and machine capability. However, ACO systems rely on process models to define the optimal conditions, so they are still affected by modeling inaccuracy and noise. This paper presents the method of Recursive Constraint Bounding (RCB) which is designed to cope with modeling uncertainty as well as process time-variability. RCB uses a model, similar to other ACO methods. However, it considers confidence levels and noise buffers to account for degrees of inaccuracy and randomness associated with each modeled constraint. RCB assesses optimality by measuring the slack in individual constraints after each part is completed (cycle), and then redefines the constraints to yield more aggressive machine settings for the next cycle. In this paper, enhancements to RCB are presented and demonstrated in application to internal cylindrical plunge grinding.

1 INTRODUCTION

Enhancing productivity is a continuing objective in manufacturing research. Typical measures of manufacturing productivity are production time or cost, and/or part quality. Approaches that have been proposed to improve these measures attempt to regulate measurements of process behavior and part quality by adjusting machine settings in response to new measurements as they become available from the process. For processes where analytical models have been developed, these models are used as the basis for adjusting the machine settings.

Model based approaches to enhancing process efficiency rely on a previously developed analytical model to provide a mapping between machine settings and measurements that can be used to select machine settings by solving a traditional constrained optimization problem [4, 9]. However, traditional optimization methods are hindered by modeling uncertainty. Sources of modeling uncertainty in manufacturing processes are (1) the diversity of manufacturing conditions due to variations in material properties, tool type, and lubrication, (2) the stochastic nature of these processes caused by material inhomogeneity, workpiece misalignment, and measurement noise, and (3) their time-variability due to tool wear. Since analytical models do not generally account for these sources of uncertainty, traditional optimization methods are unlikely to result in acceptable part quality.

The Recursive Constraint Bounding (RCB) methodology relies on concepts from adaptive control and optimization. In adaptive control, the distances from part quality measurements to their specifications are used to characterize the state of the process. In optimization, a sub-problem is set up and solved in terms of information derived from the process using techniques such as finite perturbation and regression. The unique contribution of RCB is that it combines these two concepts by adapting the optimization sub-problem based on the distances of the part quality measurements from their specifications. This paper addresses enhancements to the RCB methodology for improving robustness, functionality, and flexibility.

2 BACKGROUND

Perhaps the simplest model-based approach that has been applied to regulation of process behavior and part quality in manufacturing processes is Adaptive Control with Constraints (ACC). In ACC, power or cutting force is regulated at a specified level by adjusting machine settings [3, 11, 14, 16]. Although ACC can avoid interruptions in the cut due to tool breakages in machining, or safeguard against thermal damage (burn) to the workpiece in grinding, it is not explicitly designed to improve process efficiency in terms of production cost or time. The Adaptive Control approach which explicitly addresses process efficiency is referred to as Adaptive Control Optimization (ACO) [1]. In ACO, the machine settings are adapted so as to minimize production cost or cycle time while satisfying constraints in response to part and/or process feedback. This interactive approach to process regulation enables the ACO systems to cope with modeling uncertainty, which is a primary factor hindering regulation of manufacturing processes.

The earliest attempt at ACO was the Bendix System [2], where the machining removal rate was continually maximized through changes in the feedrate and spindle speed in response to feedback measurements of cutting torque, tool temperature, and machine vibration. However, the Bendix System was limited in applicability due to the need to estimate tool wear based on an accurate model. A subsequent advancement in ACO was the Optimal Locus Approach [1, 10], which made it possible to forego estimation of tool wear. In this approach, the locus of the optimal points associated with various levels of tool wear is computed, and the true optimal point is sought where process and part quality constraints become tight. The Optimal Locus Approach can avoid estimation of tool wear by using the tightness of constraints as the measure for optimality. However, it still relies on the accuracy of the process model for computing the optimal locus and determining 'a priori' which constraints are tight at the optimum. Since the success of this approach depends on the premise that modeling uncertainty has a negligible effect on the accuracy of the optimal locus, it will produce sub-optimal results when this premise is invalid.

The first ACO method based on RCB was RCB_1 [5, 6] which was designed to interact closely with a nonlinear program throughout the search process so as to ensure constraint satisfaction. Given the prospective values of machine settings, RCB_1 supplies the most conservative values of the uncertain parameters of the model for each iteration of the nonlinear program. It relies upon specifications of bias limits for individual uncertain parameters of the model and rules derived from a monotonicity analysis [15] to determine the conservative parameter values. Although RCB_1 is effective in achieving minimum production time within a few cycles, it needs to be customized for individual applications, which limits its applicability to new processes.

The second ACO method based on RCB was RCB_2 [7, 8] which was developed to overcome the difficulties posed by modeling uncertainty. Like the Optimal Locus Approach and RCB_1, RCB_2 assesses optimality from the tightness in the constraints using measurements of process and part quality after each workpiece has been finished (cycle). It also uses the model of the process to find the optimal point. However, RCB_2 is significantly more straightforward to apply than RCB_1, since it only requires confidence levels associated with each uncertain constraint, as opposed to bias limit specifications for each uncertain model parameter. Furthermore, RCB_1 updates its analysis of the model between iterations of the nonlinear program, whereas RCB_2 does not. Therefore, RCB_1 is much more difficult to apply, since it must be integrated into the nonlinear programming algorithm. In contrast to the Optimal Locus Approach, RCB_2 assumes the model to be uncertain when determining which constraints are to be tight at the optimum and selecting the machine settings for each process cycle. It obtains the machine settings by solving a customized nonlinear programming (NLP) problem, and allows for uncertainty by incorporating conservatism into the NLP problem.

Under deterministic conditions (no modeling uncertainty), the NLP problem would yield the optimal machine settings for the process. In practice, however, the optimal point of the model differs from that of the process, due to inherent modeling inaccuracies and randomness associated with constraints. As such, there is a strong possibility that the optimal point of the model will violate the process and part quality constraints, thus producing scrap. In order to avoid constraint violation, a recursive approach to constraint tightening (bounding) is adopted in RCB_2. The distance from the constraint measurements of the cycle just completed to the absolute limit of the constraint is defined as the slack in each constraint. The NLP problem is then formulated so as to minimize the objective function (usually cycle-time or cost) while removing a portion of these slacks, thus yielding more aggressive machine settings for the next cycle. In RCB_2, the slack portions removed for each cycle are defined in terms of the confidence levels and noise buffers which account for the inaccuracy and randomness, respectively, of individual modeled constraints. The consideration of separate confidence levels and noise buffers for individual constraints in RCB_2 enables the convergence of individual constraints to be tailored according to the severity of modeling uncertainty associated with each constraint. The repeated minimization of the objective function with progressively smaller slacks leads to tight constraints and optimal machine settings.

This paper addresses enhancements to the RCB methodology for improving robustness, functionality, and flexibility. These enhancements include new techniques for estimating the probability of constraint satisfaction and automating the computation of noise buffers and the capability to set convergence criterion in terms of maximum constraint violation probabilities. In this paper, these enhancements to Recursive Constraint Bounding are demonstrated in cycle-time reduction of cylindrical plunge grinding.

3 RCB METHODOLOGY

In this section, enhancements to Recusive Constraint Bounding (RCB) are developed. In RCB, the optimization sub-problem is formulated with a fixed model and adapted based on measurements of part quality for each part. The basic methodology is described in subsection 3.1, and the enhancements are presented in subsection 3.2.

3.1 Basic Methodology

Optimization of a machining process can be considered as a constrained nonlinear programming (NLP) problem where the machine settings correspond to the control variables, and the process and part quality measurements to the constraints. In general, a constrained NLP problem is defined as [12]:

$$\text{minimize} : \quad f(\mathbf{x}) \tag{1}$$

$$\text{subject to} : \quad \mathbf{g}(\mathbf{x}) \leq 0 \tag{2}$$

$$\mathbf{h}(\mathbf{x}) = 0 \tag{3}$$

$$\mathbf{x}_{LB} \leq \mathbf{x} \leq \mathbf{x}_{UB} \tag{4}$$

where $f(\mathbf{x})$ represents the objective function, $\mathbf{x} = [x_1, \cdots, x_n]$ denotes the vector of machine settings, $\mathbf{g}(\mathbf{x}) = [g_1(\mathbf{x}), \cdots, g_m(\mathbf{x})]$ and $\mathbf{h}(\mathbf{x}) = [h_1(\mathbf{x}), \cdots, h_p(\mathbf{x})]$ constitute the vectors of inequality and equality constraints, respectively, and $\mathbf{x}_{LB}$ and $\mathbf{x}_{UB}$ represent the lower and upper bounds of the machine settings, respectively. For machining processes, the objective function $f(\mathbf{x})$ usually represents cycle-time or cost, and the constraints are associated with part quality and/or machine limitations. Conventional optimization algorithms generally assume the constraints to be deterministic functions of the control variables. However, the constraints in manufacturing processes are more accurately represented as random variables whose probability distributions are functions of the control variables and are time-variant. The salient feature of Recursive Constraint Bounding is the ability to efficiently manipulate the constraints' probability distributions.

RCB relies on the premise that analytical models of machining processes are of the correct form, although they may be imprecise. As such, RCB is designed to take advantage of the form of the relationships provided by these models, but to compensate for their inaccuracies using measurements of process behavior and part quality as feedback. The basic role of RCB is to assess the optimality of the process after each cycle from the measurements of process and part quality and change the machine settings for the next cycle (see Fig. 1). RCB obtains the machine settings by solving a NLP problem that has been customized for each cycle. These customized

NLP problems are obtained by redefining the inequality constraints (Inequality (2)) as

$$\hat{\mathbf{g}}(\mathbf{x}(j)) \leq \hat{\mathbf{g}}(\mathbf{x}(j-1)) - \mathbf{c}[\mathbf{g}(\mathbf{x}(j-1)) + \mathbf{n}] \tag{5}$$

to lead to a more aggressive set of machine settings when used as the basis for nonlinear optimization. Equation (5) redefines the upper limit of the inequality constraints for the current cycle $\hat{\mathbf{g}}(\mathbf{x}(j))$ in terms of the previous modeled constraint value $\hat{\mathbf{g}}(\mathbf{x}(j-1))$, the previous measured constraint value $\mathbf{g}(\mathbf{x}(j-1))$, the confidence levels $\mathbf{c}$ and the noise buffers $\mathbf{n}$, representing the allowable changes for the individual modeled constraints. Assuming that the process is initiated with conservative machine settings that satisfy the actual constraints, the confidence levels and noise buffers control how much the nonlinear program should tighten the constraints from one iteration to the next.

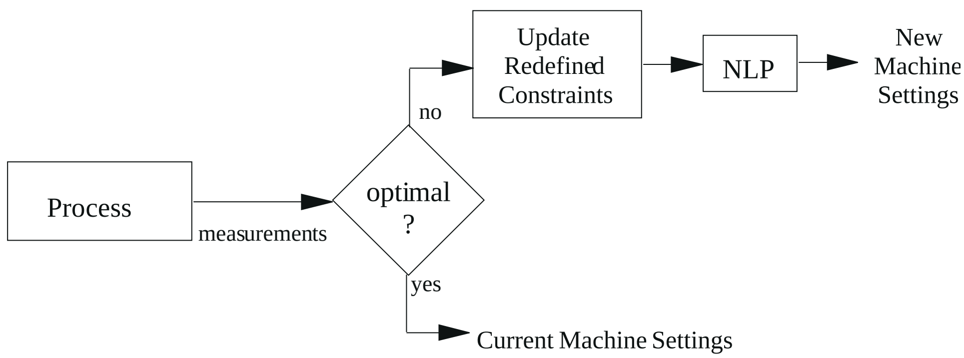

Figure 1: Schematic of RCB.

In order to clarify how the constraint redefinition for RCB was developed, consider that the value of the constraints cannot be accurately determined from the process model (i.e., $\mathbf{g}(\mathbf{x}) \neq \hat{\mathbf{g}}(\mathbf{x})$) due to modeling inaccuracies and randomness. Therefore, machine settings that would minimize the objective function while satisfying $\hat{\mathbf{g}}(\mathbf{x}) \leq 0$ do not necessarily ensure $\mathbf{g}(\mathbf{x}) \leq 0$. In machining, it is generally possible to select conservative settings that satisfy the constraints. After the process is initiated with such settings, RCB selects the machine settings such that the objective function will be reduced without violating the constraints.

In order to ensure constraint satisfaction, the machine settings for the next cycle $\mathbf{x}(j)$ need to be selected such that $\mathbf{g}(\mathbf{x}(j)) \leq 0$. However, the only information available to RCB is in the form of the model and constraint measurements from the cycle just completed. Therefore, the redefined constraints that replace Inequality (2) need to be formulated in terms of $\hat{\mathbf{g}}(\mathbf{x}(j))$ as

$$\hat{\mathbf{g}}(\mathbf{x}(j)) \leq \mathbf{U} \tag{6}$$

The main contribution of RCB is its definition of this upper bound such that it is robust to modeling inaccuracy and randomness in the constraint values. As was stated earlier, RCB relies on the premise that the model of the process correctly represents its form. Based on this premise, the assumption is made here that this model approximately represents the changes in the constraints due to changes in the machine settings, as

$$\mathbf{g}(\mathbf{x}(j)) - \mathbf{g}(\mathbf{x}(j-1)) \simeq \hat{\mathbf{g}}(\mathbf{x}(j)) - \hat{\mathbf{g}}(\mathbf{x}(j-1)) \tag{7}$$

Although modeling inaccuracy and randomness prevent RCB from directly using the above equation for redefining the constraints, it provides the basis for relating $\mathbf{g}(\mathbf{x}(j))$ to $\mathbf{g}(\mathbf{x}(j-1))$, as well as to $\hat{\mathbf{g}}(\mathbf{x}(j))$ and $\hat{\mathbf{g}}(\mathbf{x}(j-1))$ which are available to RCB from the model.

In RCB, allowance for randomness is provided by noise buffers, $\mathbf{n} = [n_1, ..., n_m]$, which define the width of the noise distributions of $\mathbf{g}(\mathbf{x})$. If adequate constraint measurements are available, the noise buffer n_i can be obtained as

$$n_i = k_i s_i \tag{8}$$

where s_i represents the standard deviation of the constraint measurements and k_i denotes a constant typically between 6 and 12. The noise buffer n_i can alternatively be estimated based on experience if adequate constraint measurements are unavailable. In order to explain how the noise buffers are utilized to establish upper bounds on the constraints, let us consider a case where the machine settings for the next cycle are very close to the settings for the cycle just completed, that is $\mathbf{x}(j) = \mathbf{x}(j-1) + \epsilon \simeq \mathbf{x}(j-1)$. For this case, the upper bounds on the actual constraint values can be defined as

$$\mathbf{g}(\mathbf{x}(j-1)+\epsilon) - \mathbf{g}(\mathbf{x}(j-1)) \leq [\hat{\mathbf{g}}(\mathbf{x}(j-1)+\epsilon) - \hat{\mathbf{g}}(\mathbf{x}(j-1))] + \mathbf{n} \tag{9}$$

This inequality provides an upper bound on the change in the constraint measurements, but it is limited to infinitesimal changes in the machine settings. In cases where $\mathbf{x}(j) \neq \mathbf{x}(j-1) + \epsilon$, modeling inaccuracy could result in changes in the constraint measurements that are larger than $[\hat{\mathbf{g}}(\mathbf{x}(j)) - \hat{\mathbf{g}}(\mathbf{x}(j-1))] + \mathbf{n}$. In order to extend Inequality (9) so that larger changes in the machine settings can be accommodated, confidence levels $\mathbf{c} \in [0, 1]$ are introduced on the right hand side of inequality (9) as

$$\mathbf{g}(\mathbf{x}(j)) - \mathbf{g}(\mathbf{x}(j-1)) \leq \frac{1}{\mathbf{c}}[\hat{\mathbf{g}}(\mathbf{x}(j)) - \hat{\mathbf{g}}(\mathbf{x}(j-1))] + \mathbf{n} \tag{10}$$

to account for the inaccuracy of individual modeled constraints. With the inclusion of the confidence levels, the upper bounds established in terms of the modeled values of constraints (right hand side of Inequality (10)) can be made sufficiently large so as to account for modeling inaccuracy associated with individual constraints. Accordingly, smaller confidence levels can be selected for constraints that are less accurately represented by the model so that a larger upper bound will be placed on the changes in the constraints.

While Inequality (10) defines the upper bound on the actual constraint changes, it does not provide the upper bound on $\hat{\mathbf{g}}(\mathbf{x}(j))$ ($\mathbf{U}$ in Inequality (6)) that is needed for redefinition of the NLP problem. In order to develop this upper bound, we note that the absolute requirement in the NLP problem is $\mathbf{g}(\mathbf{x}(j)) \leq 0$. This is equivalent to

$$\mathbf{g}(\mathbf{x}(j)) - \mathbf{g}(\mathbf{x}(j-1)) \leq 0 - \mathbf{g}(\mathbf{x}(j-1)) \tag{11}$$

which defines the absolute limit on changes in the actual constraints. Satisfaction of this absolute limit in light of Inequality (10) is ensured when

$$\frac{1}{\mathbf{c}}[\hat{\mathbf{g}}(\mathbf{x}(j)) - \hat{\mathbf{g}}(\mathbf{x}(j-1))] + \mathbf{n} \leq 0 - \mathbf{g}(\mathbf{x}(j-1)) \tag{12}$$

which states that the upper bound for Inequality (10) must be less than or equal to the upper bound for Inequality (11). Inequality (12) provides the basis for defining the upper limit on $\hat{\mathbf{g}}(\mathbf{x}(j))$ ($\mathbf{U}$ in Inequality (6)) so that constraint satisfaction is guaranteed. Rearranging Inequality (12) yields

$$\hat{\mathbf{g}}(\mathbf{x}(j)) \leq \hat{\mathbf{g}}(\mathbf{x}(j-1)) - \mathbf{c}[\mathbf{g}(\mathbf{x}(j-1)) + \mathbf{n}] \tag{13}$$

which defines the upper bound for $\hat{\mathbf{g}}(\mathbf{x}(j))$ in terms of the modeled constraints and their measured values from the cycle just completed. Inequality (13), which is identical to Inequality (5), represents the redefined constraints to be used in the customized NLP problem in place of Inequality (2). Note that under deterministic conditions (accurate model, without noise), the modeled constraint values $\hat{\mathbf{g}}(\mathbf{x})$ and their measured values $\mathbf{g}(\mathbf{x})$ would be identical, the confidence levels would be assigned the value of 1 (accurate model) and the noise buffers would have the value of 0 (noise-free conditions). Under these conditions, the right hand side of Inequality (13) would reduce to zero, and Inequality (13) would be equivalent to Inequality (2).

The conceptual basis of RCB's design is illustrated in Fig. 2. The dark and light data points in this figure represent measured and modeled values of a constraint for successive cycles, respectively, and the dotted arrows point to the upper limit of the constraint in successively redefined NLP problems. The top of the gray area represents the allowable limit of a constraint, and the bottom of this area denotes the limit when noise is taken into consideration. Note that the width of the gray area is the value of the noise buffer. When the distance from a particular measurement to its limit is less than its noise buffer (data point within the gray area) the constraint cannot be safely tightened. In such cases, the value of $c[g(\mathbf{x}(j-1)+n]$ is set to zero (e.g., Cycle 6) signifying that the modeled constraint value should not be changed. When the distance from a particular measurement to its limit is greater than its noise buffer (data point outside the gray area) the constraint is tightened using Inequality (13). In such cases, the distance from each constraint measurement (data point) to its upper limit represents the slack in the constraint ($0 - \mathbf{g}(\mathbf{x}(j-1))$ in Inequality (12)), and the dotted arrows represent the portion of the slack ($-\mathbf{c}[\mathbf{g}(\mathbf{x}(j-1)) + \mathbf{n}]$) that RCB attempts to remove by redefining the NLP problem. The actual change in the slack may be less than the desired change (Cycle 3) or greater than the desired change (Cycle 4). As such, if the confidence level were assigned the value of 1 (causing the dotted arrow to point to the bottom of the gray area) the actual constraint may fall above the gray area and result in constraint violation. Assigning a value less than one to the confidence level provides a safety margin to improve the likelihood of constraint satisfaction. This improvement, however, is provided at the cost of reducing the rate of convergence to the optimum, as discussed in [7].

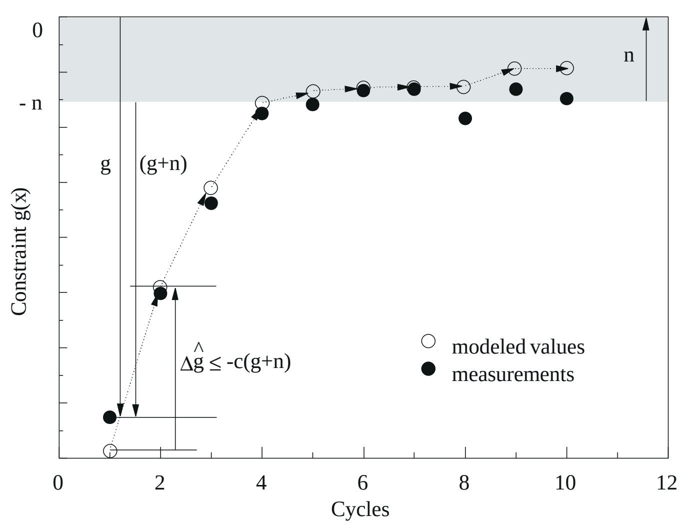

Figure 2: Constraint tightening in RCB.

As the NLP problem is repeatedly redefined and solved, the machine settings approach their optimal values and the process and part quality measurements approach their respective limits. At the steady state, some slack may remain in the constraints due to the conservative estimates of the noise buffers, $\mathbf{n}$. After all of the constraint measurements have converged within these conservative noise buffers, the process can be repeated to obtain more constraint measurements for improving the estimates of the noise buffers using Eq. (8). In cases where the new noise buffer estimates are smaller than their original values, the NLP problem can be redefined with the new noise buffers so as to further tighten the constraints and reduce the objective function.

3.2 RCB Methodology Enhancements

The enhancements to RCB are designed to improve robustness, functionality, and flexibility. Robustness is improved by requiring that the noise buffers be computed from the standard deviations of the estimated probability distributions of the constraints instead of allowing them to be approximated based on the experience of the machine operator. This improves RCB's control over the likelihood of constraint violation during and after optimization at the cost of an increase in the number of process cycles. Functionality is improved by explicitly specifying the sample size for estimating the constraints' probability distributions. Previously, the noise buffers were re-computed after all constraints were tightened using conservative noise buffers based on operator expertise. The size of the sample for re-computing the noise buffers was determined by how many samples continued to fall within the conservative noise buffers, so accurate estimations of the noise buffers could only obtained at the end of optimization and only if the operator chose to continue measuring part quality. With the enhancements presented here, the sample size is automatically increased or decreased throughout optimization as needed to balance the need to ensure constraint satisfaction against the need to reduce the objective function. Flexibility is improved by setting the maximum allowable probability of constraint violation for each constraint individually. Thus, the corresponding noise buffers are estimated as conservatively or aggressively as needed. From these probabilities, RCB automatically calculates the corresponding sample size and noise buffers. This requires that the sample size be sufficiently large to provide adequate accuracy when estimating the noise buffers and their corresponding constraint satisfaction probabilities.

As stated above, the noise buffers are now computed from the standard deviations of the constraints' estimated probability distributions as

$$\mathbf{n} = \mathbf{k} \cdot \mathbf{s} \tag{14}$$

where s represents the sample standard deviation. The probability distributions are estimated from the constraint measurements for a sample of process cycles with identical machine settings. Initially, RCB uses 10 process cycles per sample, since the objective function has not been reduced yet. Once estimates of the noise buffers have been obtained, RCB uses one process cycle per sample until all of the constraint measurements are within their respective noise buffer regions (gray area in Figurer 2) or until a constraint is violated. If a constraint is violated, RCB retreats to the previous machine settings, collects a 10 cycle sample of constraint measurements, and re-computes the noise buffers. If a constraint is violated during this sample, RCB proceeds as if the constraint were satisfied except that the corresponding confidence level is temporarily set to 1.0, signifying that RCB should reduce the constraint by the measured violation plus the noise buffer in order to bring the bulk of the constraint's probability distribution below the absolute limit of the constraint. If the confidence level were not adjusted, the probability of constraint violation in subsequent cycles would be unacceptably high. Table 1 summarizes the steps used in the application of RCB with the enhancements presented here.

4 GRINDING INVESTIGATION

The effectiveness of RCB is illustrated in simulation for internal cylindrical plunge grinding, where six machine settings are adjusted to satisfy three inequality constraints and one equality constraint.

In cylindrical grinding, material is removed from the internal cylindrical surface by feeding a grinding wheel that is rotating at a high speed into the workpiece which rotates at a much lower speed (see Fig. 3). The infeed control cycle $\mathbf{u} = [u_1, u_2, u_3]$ is typically characterized by three successive stages as illustrated in Fig. 4: (1) roughing with a relatively fast infeed velocity u_1, (2) finishing with a slower infeed velocity u_2, and (3) spark-out at zero infeed velocity (u_3=0). This is followed by rapid retraction to disengage the wheel from the workpiece.

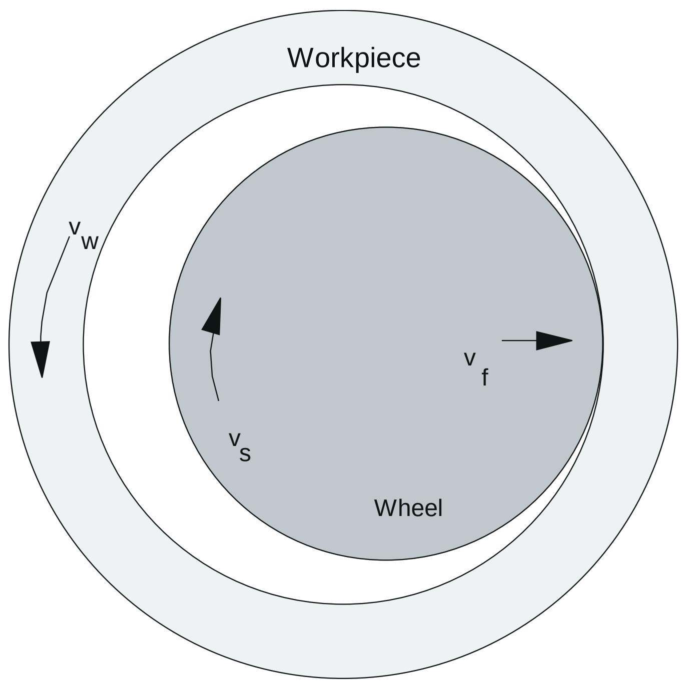

Figure 3: Diagram of an internal cylindrical plunge grinding operation.

In response to the controlled infeed, the radial size reduction of the workpiece follows the actual infeed curve as shown in Fig. 4. The transient in the actual infeed at the beginning of each stage is attributed mainly to the elastic deflection of the system and to the radial wear of the grinding wheel. This transient behavior can be approximated by a first order system characterized by a time constant [13].

The constraint measurements were simulated using a grinding model defined as [17]:

$$\begin{aligned} \text{Minimize cycle-time:} \quad & T = t_1 + t_2 + t_3 \qquad (15) \\ \text{with respect to:} \quad & u_1, u_2, t_1, t_2, t_3, s_d \end{aligned}$$

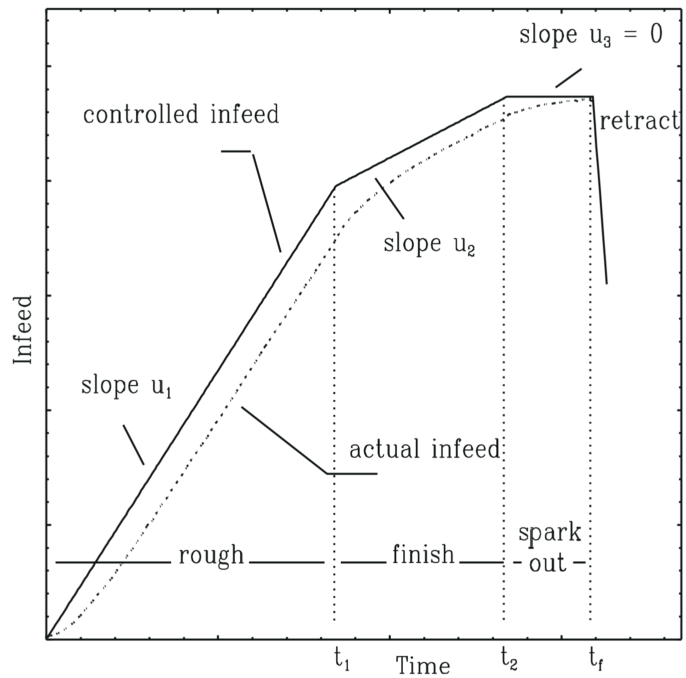

Figure 4: Illustration of a grinding cycle consisting of roughing, finishing and spark-out stages.

subject to :

$$g_1 = z_1 - q_2 \leq 0 \quad \text{(burning constraint)} \tag{16}$$

$$g_2 = R_m - R_{max} \leq 0 \quad \text{(surface finish constraint)} \tag{17}$$

$$g_3 = r - r_{max} \leq 0 \quad \text{(out-of-roundness constraint)} \tag{18}$$

$$h_1 = u_1 t_1 + u_2 t_2 - \Delta r = 0 \quad \text{(size constraint)} \tag{19}$$

The objective function is the total cycle-time T, which is defined as the sum of the times for the three successive infeed stages, $[t_1, t_2, t_3]$. The machine settings are the stage times $[t_1, t_2, t_3]$, the programmed infeed rates for the first two stages $[u_1, u_2]$, and the dressing lead s_d. For these simulations, the wheel was dressed after each cycle using a single point diamond dresser. The dressing lead s_d, which specifies the crossfeed per revolution of the wheel, determines the initial sharpness of the wheel. The relationships among the constraints and machine settings are given in the Appendix.

Minimization of the total cycle-time requires that tradeoffs among the three stage times be balanced through an examination of their relationships with the various constraints using Eqs. (16) - (19) and Eqs. (20) - (40) in the Appendix. The burning constraint in Inequality (16) requires that the thermally damaged (burned) layer on the workpiece due to excessive grinding temperatures during the roughing stage be completely removed during the subsequent finishing stage. As such, a deeper layer of thermally damaged material (depth of burn z_1) caused by a more aggressive roughing infeed rate u_1 can be balanced by increasing the depth of material removal in the finishing stage q_2. An alternative to this burning constraint is to completely avoid thermal damage during the roughing stage, which is more restrictive but may be desirable for grinding of critical components [17]. Inequality (17) defines the surface finish constraint, where R_m denotes the measured surface roughness and R_{max} its maximum allowable value. Inequality (18) defines the out-of-roundness constraint, where r represents the out-of-roundness value and r_{max} the maximum allowable out-of-roundness. Equality (19) defines the size requirement, where Δr denotes the radial depth of material to be removed.

Many of the coefficients and exponents in the equations included in the Appendix were empirically obtained under specific operating conditions. Under different operating conditions, the values of these coefficients and exponents would be *biased*. In order to demonstrate RCB's ability to cope with modeling bias, some of the model coefficients and exponents were biased by a random factor between $+/-10\%$ of their original values. Similarly, the simulation equations were modified to include noise in the constraint equations by multiplying the constrained variables by a random factor between $+/-10\%$ of their original values. As such, the simulation differs from the model used by RCB in two ways. First, the simulation uses biased parameters whereas the model uses their nominal values. Second, the simulated constraint values are randomly perturbed at each constraint evaluation, whereas the model will calculate identical constraint values for identical machine settings.

The arithmetic average surface roughness constraint and out-of-roundness constraint were selected as 0.7μm and 0.6μm, respectively. In these simulations, AISI52100 hardened steel bearing workpieces with an internal diameter d_w of 70mm and width b of 9mm were machined using a 32A80M6VBE grinding wheel with an external diameter d_s of 50mm. The peripheral speeds of the wheel v_s and the workpiece v_w were 37m/s and 0.55 m/s, respectively.

5 GRINDING RESULTS

In this section, the enhancements presented in Section 2 are demonstrated using several examples of RCB simulation runs. The first two examples demonstrate the advantage of RCB's use of variable sample sizes. The first example was run with a constant sample size of 30. The second example was run with sample sizes of 1 and 30. For iterations where the noise buffers were estimated a sample size of 30 was used, whereas a sample size of 1 was used for iterations where noise buffers were not estimated. The resulting cycle times and constraints are shown in Figs. 5 and 6. As can be seen in Figs. 5 and 6, the cycle time reduction and constraint tightening curves are virtually identical, which demonstrates RCB's ability to tighten the constraints without obtaining samples of the constraints for estimating the standard deviations of their probability distributions.

The next two examples demonstrate RCB's ability to recuperate after a constraint violation occurs. In the third example, the initial sample size for estimating the constraints' probability distribution is set to ten. In this particular case (one of many such simulation runs) a small constraint violation occured when the constraints were almost fully tightened. This constraint violation happened because the standard deviation of the constraint's probability distribution was underestimated. As such, when RCB tightened the constraints, the noise buffer was too small and the constraint was over-tightened. The constraint tightening curves for this example is given in Fig. 7. As can be seen in Fig. 7, RCB adjusts the machine settings so that the constraint's distribution is again within its allowable limit.

In the fourth and final example, RCB is given an initial point which results in a constraint violation. The constraint measurements for this example are given in Fig. 8, which shows that once again, RCB is able to recover from the constraint violation and continues with constraint tightening (and cycle-time reduction, which is not shown).

The results presented in this section demonstrate the efficacy of RCB as a methodology for

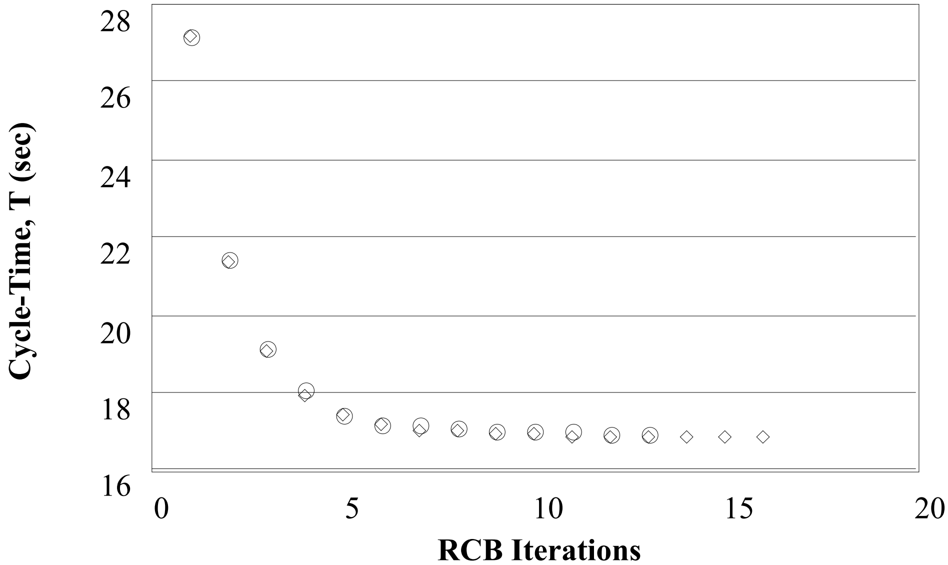

Figure 5: Cycle-time reduction by RCB for examples 1 and 2.

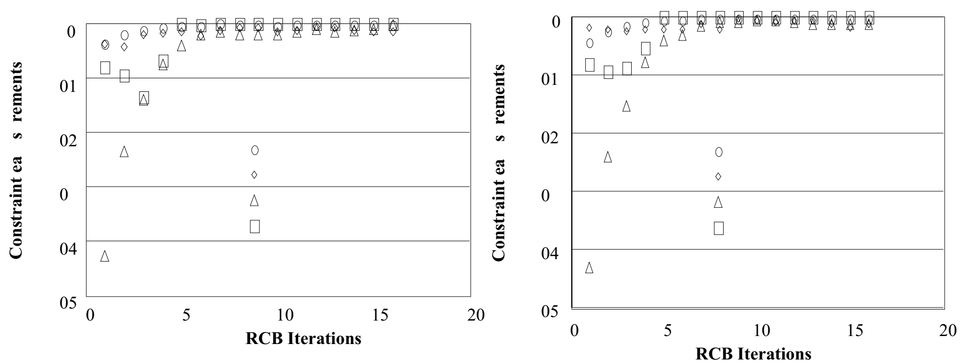

Figure 6: Constaint tightening by RCB for examples 1 and 2.

robust optimization of a manufacturing process subject to modeling uncertainty. Cycle-time is reduced for internal cylindrical plunge grinding while satisfying constraints despite the presence of noise and modeling bias.

6 CONCLUSION

In this paper, enhancements to Recursive Constraint Bounding have been developed for improving robustness, functionality, and flexibility. RCB has been shown to be an effective methodology for adjusting the machine settings from cycle to cycle in order to reduce cycle-time. RCB is designed to cope with modeling uncertainty and process time-variability due to tool wear, and can be used either as a selection guide to the machine operator or as the basis of a supervisory module for production control. Since RCB uses separate confidence levels and noise buffers for each constraint, incorporating additional machine inputs or constraints does not result in a combinatorial increase in computation time or convergence iterations. æ

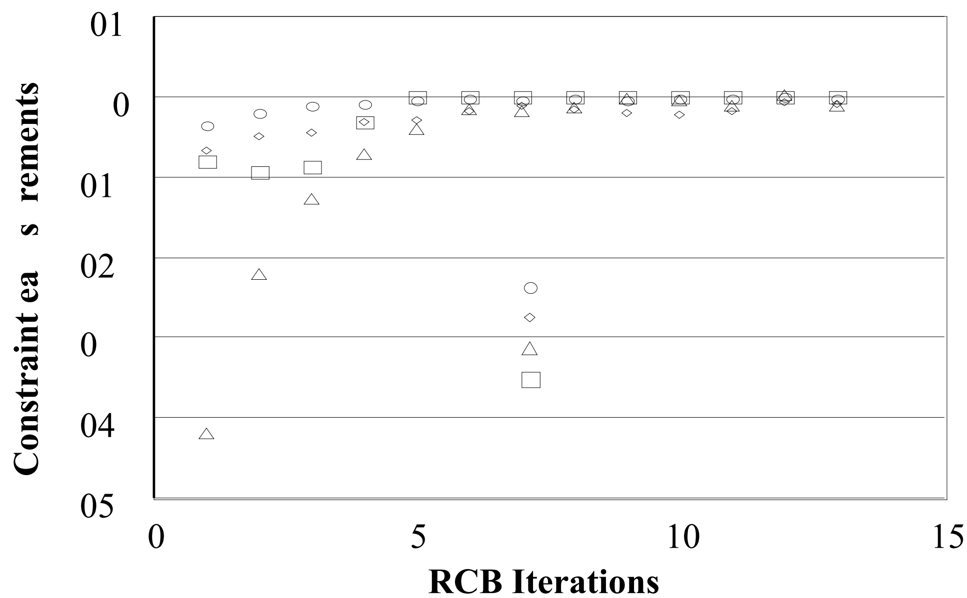

Figure 7: Constaint tightening by RCB for example 3.

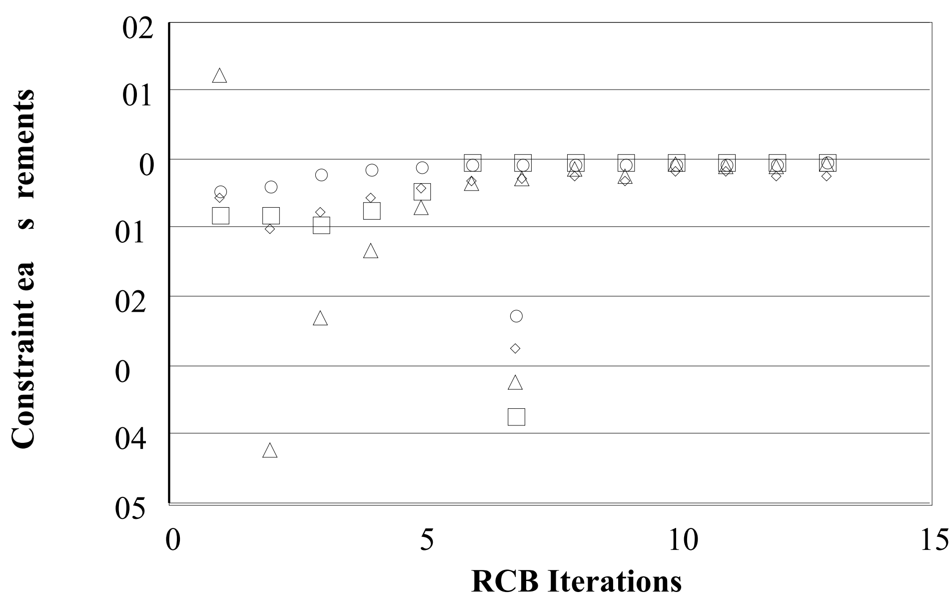

Figure 8: Constaint tightening by RCB for example 4.

Table 1: Algorithm for application of RCB with enhancements.

I. Setup optimization sub-problem
- A. Define objective function
- B. Define and model inequality and equality constraints
- C. Define $\mathbf{x}_{LB}$ and $\mathbf{x}_{UB}$

II. Setup RCB
- A. Select conservative initial machine settings $\mathbf{x}(1)$
- B. Repeatedly run process with $\mathbf{x}(j)$ until initial sample size is reached and compute the noise buffers $\mathbf{n}$
- C. Assign conservative values to the confidence levels $\mathbf{c}$
- D. Set j to 1

III. Run process
- A. Set sample iteration = 1
- B. Run process with $\mathbf{x}(j)$ to obtain $\mathbf{g}$(j)
- C. If sample iteration < sample size
  - 1. set sample iteration = sample iteration +1
  - 2. go to III.B.
- D. Else if sample size > 1,
  - 1. compute noise buffers as $\mathbf{n} = \mathbf{k} * \mathbf{s}$
  - 2. set sample size = 1
- E. Assess Optimality
  - 1. If $(0 - \mathbf{n}) \leq \mathbf{g} \leq 0$
    - a. if (sample size < max sample size)
      - i. set sample size = sample size +10
      - ii. go to III.A.
    - b. else end optimization
  - 2. Else if $\mathbf{g} < 0$
    - a. update redefined constraints
      $\hat{\mathbf{g}}(\mathbf{x}(j)) \leq \hat{\mathbf{g}}(\mathbf{x}(j-1)) - \mathbf{c}[\mathbf{g}(\mathbf{x}(j-1)) + \mathbf{n}]$
    - b. goto IV
  - 3. Else $(g > 0)$
    - a. set sample size = 10
    - b. go to III.A.

IV. Run NLP
- A. Obtain $\mathbf{x}(j+1)$
- B. Set j to $j+1$
- C. Goto III

APPENDIX

GRINDING RELATIONSHIPS

The following equations obtained from grinding theory are used for calculating and estimating the process parameters [17]:

$$a = v_f/n_w \tag{20}$$
$$n_w = v_w/(\pi d_w) \tag{21}$$
$$d_e = (d_s d_w)/(d_w - d_s) \tag{22}$$
$$v_i = u_i^{'} - (u_i^{'} - v_{i-1}) exp(-\frac{t_i}{\tau}) \tag{23}$$
$$q_i = u_i^{'} t_i + \tau[(u_i^{'} - v_{i-1}) exp(-\frac{t_i}{\tau}) - (u_i^{'} - v_{i-1})] \qquad i = 1, 2, 3 \tag{24}$$
$$u_i^{'} = u_i/f \tag{25}$$
$$f = 1 + d_w/(d_s G) \tag{26}$$
$$\begin{aligned} P = {} & .0138\pi d_w v_f + 9.62 \times 10^{-7} v_s + \\ & [8.55 \times 10^{-6} + 2.10 v_w/(v_s d_e)] v_s v_w^{-.5} (\pi d_w v_f d_e)^{.5} A_{eff} \end{aligned} \tag{27}$$
$$P_b = 0.00617\pi d_w v_f + 0.0072(\pi d_w d_e v_f v_w)^{0.25} \tag{28}$$
$$z = -1.449(\frac{v_w l_c}{4\alpha})^{0.37} \frac{2\alpha}{v_w} ln[\frac{\pi k l_c \theta_{mb} v_w}{6.2\alpha\pi d_w v_f (\frac{v_w l_c}{4\alpha})^{0.53}(u - 0.45u_{ch})}] \tag{29}$$
$$u = P/(\pi d_w v_f) \tag{30}$$
$$l_c = (\pi d_w d_e v_f / v_w)^{0.5} \tag{31}$$
$$r = \pi g_{n_3} k_3 v_3 d_w / v_w \tag{32}$$
$$R = g_{n_2} R_o S_d^x a_d^y (\frac{\pi d_w v_1}{v_s})^{k_1} [1 + exp(k_2 t_3)] \tag{33}$$
$$R_f = 1.5R \tag{34}$$
$$R_a = [R_f + (R_o - R_f) exp(\frac{n_r - 1}{n_o - 1})] \tag{35}$$
$$A_{eff} = g_{n_1}(-0.008) A_0 ln(1.4 \times 10^4 m\delta) + 5.299 \times 10^{-4} L_s \tag{36}$$
$$L_s = \left[\frac{u_1'(1 - exp(\frac{-t_1}{\tau}))}{2.5}\right]^{0.5} \frac{v_s}{(\pi^2 0.25 d_w d_s^{0.5})} \tag{37}$$
$$\delta = 1.1 \times 10^{-11} a_d^{0.75} s_d^{1.75} \tag{38}$$
$$\tau = f(u_1 t_1 - f q_1)/u_1 \tag{39}$$
$$G = (\pi v_1 d_w)/(\pi w_1 d_s) = (v_1 d_w)/[(u_1 - v_1) d_s] \tag{40}$$

a — depth of cut
a_d — dressing depth
A_0 — constant
A_{eff} — effective dullness
d_e — equivalent diameter

d_w — diameter of part
d_s — diameter of wheel
f — modified coefficient
G — grinding ratio
k — thermal conductivity of part
l_c — contact length
L_s — accumulated grinding length
m — constant
n_w — rotational speed of workpiece
P — grinding power
P_b — burning power
q_i — actual infeed for the ith stage
r — actual value of out-of-roundness
R_a — measured surface finish
R_o — constant
s_d — dressing lead
t_i — grinding time for the ith stage
u — specific energy
u_{ch} — specific energy for chip formation
u_i — programmed infeed rate
$u_i^{'}$ — effective programmed infeed rate
v_f — actual infeed rate
v_i — actual infeed rate for the ith stage
v_s — velocity of wheel
v_w — velocity of workpiece
x — constant
y — constant
z — depth of burn
α — thermal diffusivity of part
β — fraction of finishing stage with burning
δ — equivalent dressing infeed angle
γ — constant
θ_{mb} — critical temperature for burning
τ — time constant

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